

Deviation from Tri-Bimaximal Mixing and Large Reactor Mixing Angle.

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Abstract

Recent observations for a non-zero θ_{13} have come from the T2K and MINOS experiments. A model of lepton mixing with a $2-3$ flavor symmetry has been considered by A. Rashed and A. Datta to accommodate the sizable θ_{13} measurement. They have discussed deviations of the Bimaximal (BM) structure from the charged lepton and neutrino sector. In this work, we extend this model to derive deviations from the tri-bimaximal (TBM) pattern arising from breaking the flavor symmetry in the neutrino sector, while the charged leptons contribution has been discussed in the previous work. Choosing a certain structure to the neutrino mass matrix under a certain condition generates the TBM form in the symmetric limit. Different flavor symmetries and particle content are required to produce the TBM, rather than the BM, as the leading order mixing. This, in turn, leads to different phenomenology. Contributions from the neutrino sector towards accommodating the non-zero θ_{13} value are obtained.

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1 Introduction

Neutrino oscillations can be parametrized in terms of three mixing angles θ_{12} , θ_{13} , θ_{23} and Dirac (δ) and Majorana (ζ_1 , ζ_2) CP violating phases

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (1)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, and $P_\nu \equiv \{1, e^{i\zeta_1}, e^{i\zeta_2}\}$ is a diagonal phase matrix, which is physically relevant if neutrinos are Majorana particles. Experiments have put no constraints on the Majorana phases, therefore, we usually ignore them. The experiments have shown that the angles of lepton mixing are relatively larger than their counterparts in the quark sector. The best fit values for the mixing angles are given as [1]

$$\theta_{12} = 34.5 \pm 1.0(^{+3.2}_{-2.8})^\circ, \quad \theta_{23} = 42.8^{+4.7}_{-2.9}(^{+10.7}_{-7.3})^\circ, \quad (2)$$

at $1\sigma(3\sigma)$ level. Recent data from the T2K [2] and MINOS [3, 4] experiments have indicated a nonzero values for θ_{13} . The results of the T2K are given at the 90% confidence level as

$$\begin{aligned} 5.0^\circ &\lesssim \theta_{13} \lesssim 16.0^\circ & (\text{NH}), \\ 5.8^\circ &\lesssim \theta_{13} \lesssim 17.8^\circ & (\text{IH}), \end{aligned} \quad (3)$$

with $\delta = 0^\circ$, where “NH” and “IH” stand for the normal and inverted neutrino mass hierarchies, respectively. The best fit values are $\theta_{13} = 9.7^\circ$ (NH) and 11.0° (IH).

The distribution of the flavors in the mass eigenstates, corresponding to the best fit values of the mixing angles, has shown that the leading order mixing method is a successful way to describe the lepton mixing. The most common patterns that have been discussed in the literatures to describe the lepton mixing, which may arise from discrete symmetries, are called; democratic (DC) [5], bimaximal (BM) [6], and tri-bimaximal (TBM) [7] mixing matrix. The realistic mixing matrix is well approximated by the tri-bimaximal pattern

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

Some previous studies have considered the TBM form in the symmetric limit of various flavor symmetries [8, 9]. Contributions from the charged lepton sector to the leptonic mixing have been studied previously [10]. Several papers have discussed the recent T2K data [11, 12]. Some of those studies have considered deviations from the charged lepton sector [11].

In Ref. [13], they have considered the decoupled 2 – 3 symmetry as the flavor symmetry of the charged lepton sector. The contributions of the charged lepton and neutrino sector have been discussed in Ref. [13] with the Bimaximal (BM) pattern being the leading order term of the lepton mixing. In this work, we assume a certain texture for the neutrino mass matrix with the third generation decoupled from the first two generations. Requiring the elements of the unitary matrix that diagonalizes the neutrino mass matrix to be independent of the mass parameters causes the leptonic mixing to have the TBM form in the symmetric limit under a certain condition.

In our model, we introduce a Lagrangian that extends the SM particle content by three right-handed neutrinos, three complex singlet scalar fields, and an additional Higgs doublet.² The symmetry group of the SM is extended by the product of the symmetries $Z_2 \times Z_4 \times U(1)$. The Z_2 symmetry serves to have a 2 – 3 symmetric Yukawa matrix in the charged lepton sector. The Z_4 yields the mass matrices in the charged lepton and neutrino sector to have decoupled structures. We present a global $U(1)$ symmetry which compels only the extra Higgs to generate the Dirac neutrino mass. Also, the $U(1)$ symmetry equates certain couplings of the right-handed neutrinos as we relate the couplings to the $U(1)$ charges. The $U(1)$ symmetry forbids the Majorana masses of the right-handed neutrinos. The Majorana neutrino masses are generated via the v.e.v of the singlet scalars and the $U(1)$ gets broken spontaneously. Without altering the lepton mixing, an additional Majorana mass term is introduced to protect one of the neutrino masses from blowing up.

Breaking the symmetry in the charged lepton sector has been studied in Ref. [13]. In the neutrino sector we generate a deviation to the TBM mixing by explicitly breaking the S_3 symmetry in the effective potential. The symmetry breaking term violates the alignment of the v.e.v's of the singlet scalar fields. The contribution of the neutrino sector to the deviation of the lepton mixing goes as $\sim \frac{v^2}{w^2}$ where v is the v.e.v of the SM Higgs and w is the scale of the v.e.v of singlet scalars. Assuming $w \sim \text{TeV}$, the deviations are sufficient to generate the realistic lepton mixing which is a hint that the TBM is not an accidental symmetry [15].

The paper is organized in the following manner: In Sec. 2 we study the TBM mixing in the flavor symmetric limit. In Sec. 3 we discuss the Lagrangian that describes the flavor symmetry in the charged lepton and neutrino sector. In Sec. 4 we break the flavor symmetry to generate the realistic leptonic mixing. In Sec. 5 we show the numerical results due to the symmetry breaking, and, finally, in Sec. 6 we summarize the results reported in this work.

² Some recent motivations for considering two Higgs doublet models can be found in Ref. [14].

2 The TBM matrix from flavor symmetry

In Ref. [13], it was assumed the Yukawa matrix of the charged lepton sector to be invariant under the $\mu - \tau$ interchange. Since the leptonic mixing matrix is composed of pure numbers, Ref. [13] naturally supposed the mass matrices to be diagonalized by a unitary matrices composed of pure numbers. This results in the Yukawa matrix of the charged lepton sector to be decoupled as

$$Y_{23}^L = \begin{pmatrix} l_{11} & 0 & 0 \\ 0 & \frac{1}{2}l_T & \frac{1}{2}l_T \\ 0 & \frac{1}{2}l_T & \frac{1}{2}l_T \end{pmatrix}. \quad (5)$$

This Yukawa matrix leads to zero muon mass $m_\mu = 0$, which will be later the source of symmetry breaking. The Yukawa matrix is invariant under a Z_2 symmetry and diagonalized by the unitary matrix W_{23}^l given by

$$W_{23}^l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (6)$$

In the neutrino sector we assume that, in the symmetric limit, \mathcal{M}_ν has a certain texture

$$\mathcal{M}_\nu = \begin{pmatrix} a & d & 0 \\ d & b & 0 \\ 0 & 0 & c \end{pmatrix}. \quad (7)$$

Here, we consider all the parameters are real. This matrix can be diagonalized by the matrix

$$W_{12}^\nu = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (8)$$

$$s_{12} \equiv \sin \theta_{12}, \quad c_{12} \equiv \cos \theta_{12},$$

where

$$\tan 2\theta_{12} = \frac{2d}{(a-b)}. \quad (9)$$

The leptonic mixing can be described, in the symmetric limit, by the matrix

$$U_{PMNS}^s = U_\ell^\dagger U_\nu, \quad (10)$$

where

$$U_\ell = W_{23}^l, \quad U_\nu = W_{12}^\nu. \quad (11)$$

If we require θ_{12} in Eq. 9 to be independent of the parameters a , b and d , then, we have $d = \pm k(a - b)$ and in particular we obtain the tri-bimaximal mixing with $k = \sqrt{2}$. By choosing the positive sign,

$$s_{12} = \frac{1}{\sqrt{3}}, \quad c_{12} = \sqrt{\frac{2}{3}}, \quad (12)$$

where \mathcal{M}_ν is given as

$$\mathcal{M}_\nu = \begin{pmatrix} a & \sqrt{2}(a-b) & 0 \\ \sqrt{2}(a-b) & b & 0 \\ 0 & 0 & c \end{pmatrix}. \quad (13)$$

We see that the neutrino mass matrix exhibits decoupling of the first two generations from the third one. The mass eigenvalues are given by

$$\mathcal{M}_\nu^d = \text{diag} (2a - b, 2b - a, c). \quad (14)$$

3 The origin of the $\mu - \tau$ symmetry in this model

The Lagrangian that describes this model will be discussed in this section. It is assumed to be invariant under the product of the symmetries $Z_2 \times Z_4 \times U(1)$. The Yukawa Lagrangian exhibits $\mu - \tau$ symmetry, which is a Z_2 symmetry. We use the see-saw mechanism to generate the neutrino masses. The particle content of the model is given as

- three left-handed lepton doublets $D_{\alpha L}$, where α is denoted by e , μ , and τ ,
- three right-handed charged-lepton singlets α_R , and
- three right-handed neutrino singlets $\nu_{\alpha R}$.

In the scalar sector, we employ

- two Higgs doublets ϕ_j with vacuum expectation value, v.e.v's, $\langle 0 | \phi_j^0 | 0 \rangle = \frac{v_j}{\sqrt{2}}$ and
- three complex singlet scalar fields ϵ_k with v.e.v's $\langle 0 | \epsilon_k^0 | 0 \rangle = w_k$, $k = 1, 2, 3$.

The symmetries of the Lagrangians are assumed as

$$\begin{aligned} Z_2 : & D_{\mu L} \leftrightarrow -D_{\tau L}, \mu_R \leftrightarrow -\tau_R, \nu_{\mu R} \leftrightarrow -\nu_{\tau R}, \\ & D_{eL} \rightarrow D_{eL}, e_R \rightarrow e_R, \nu_{eR} \rightarrow \nu_{eR}, \\ & \epsilon_1 \rightarrow -\epsilon_1, \epsilon_2 \rightarrow \epsilon_2, \epsilon_3 \rightarrow \epsilon_3, \phi_1 \rightarrow \phi_1, \phi_2 \rightarrow \phi_2, \\ Z_4 : & \nu_{eR} \rightarrow i\nu_{eR}, e_R \rightarrow ie_R, D_{eL} \rightarrow iD_{eL}, \\ & \epsilon_1 \rightarrow i\epsilon_1, \epsilon_2 \rightarrow i\epsilon_2, \epsilon_3 \rightarrow -\epsilon_3. \quad (\text{The rest of the fields remain the same.}) \end{aligned} \quad (15)$$

The most general Lagrangian invariant under the underlined symmetries

$$\begin{aligned}
\mathcal{L}_Y &= [y_1 \bar{D}_{e_L} e_R + y_2 (\bar{D}_{\mu_L} \mu_R + \bar{D}_{\tau_L} \tau_R) + y_3 (\bar{D}_{\mu_L} \tau_R + \bar{D}_{\tau_L} \mu_R)] \phi_j \\
&+ [y_4 \bar{D}_{e_L} \nu_{eR} + y_5 (\bar{D}_{\mu_L} \nu_{\mu R} + \bar{D}_{\tau_L} \nu_{\tau R})] \tilde{\phi}_j \\
&+ \frac{1}{2} y_6 \bar{\nu}_{eR} \left(\nu_{\mu R}^c \frac{(a\epsilon_1 + b\epsilon_2)}{\sqrt{2}} + \nu_{\tau R}^c \frac{(a\epsilon_1 - b\epsilon_2)}{\sqrt{2}} \right) \\
&+ \frac{1}{2} y_7 \bar{\nu}_{eR} \nu_{eR}^c \epsilon_3 + h.c.
\end{aligned} \tag{16}$$

which is similar to the Lagrangian that describes the BM structure in Ref. [13]. Here, $\tilde{\phi}_j \equiv i\sigma_2 \phi_j^*$ is the conjugate Higgs doublet. We can simplify the above Lagrangian in many ways; We can assign $U(1)$ charges, $\{\nu_{(e,\mu,\tau)R}, \epsilon_{(1,2,3)}, D_{(e,\mu,\tau)L}, (e, \mu, \tau)_R, \phi_1, \phi_2\} = \{1/3, 2/3, 4/3, 4/3, 0, -1\}$, to compel an individual Higgs to each sector. In our model we relate the couplings to the $U(1)$ charges as $y = cq$ where y is a coupling, q is a $U(1)$ charge, and c is a constant. This leads to a universal coupling to the right-handed neutrinos. Secondly, we choose to work in the basis which has the Dirac neutrino mass matrix to be diagonal. Thirdly, to reduce the number of parameters we can impose an approximate symmetry of the Lagrangian. A $SU(3)$ symmetry where the right handed singlet fields and the left handed doublet fields transform as the $SU(3)$ triplets leads to $y_4 = y_5$. The $SU(3)$ symmetry is only satisfied by the Dirac mass term for the neutrinos and is broken by the other terms in the Lagrangian. Fourthly, imposing a transformation on the right-handed charged leptons $\mu_R \leftrightarrow \tau_R$ requires $y_2 = y_3$ leading to vanishing the muon mass. Finally, we can redefine $a\epsilon_1 \rightarrow \epsilon_1$ and $b\epsilon_2 \rightarrow \epsilon_2$. We can then rewrite the Lagrangian as

$$\begin{aligned}
\mathcal{L}_Y &= [y_1 \bar{D}_{e_L} e_R + y_2 (\bar{D}_{\mu_L} \mu_R + \bar{D}_{\tau_L} \tau_R) + y_2 (\bar{D}_{\mu_L} \tau_R + \bar{D}_{\tau_L} \mu_R)] \phi_1 \\
&+ y_D [\bar{D}_{e_L} \nu_{eR} + \bar{D}_{\mu_L} \nu_{\mu R} + \bar{D}_{\tau_L} \nu_{\tau R}] \tilde{\phi}_2 \\
&+ \frac{1}{2} y \bar{\nu}_{eR} \left(\nu_{\mu R}^c \frac{(\epsilon_1 + \epsilon_2)}{\sqrt{2}} + \nu_{\tau R}^c \frac{(\epsilon_1 - \epsilon_2)}{\sqrt{2}} \right) \\
&+ \frac{1}{2} y \bar{\nu}_{eR} \nu_{eR}^c \epsilon_3 + h.c.,
\end{aligned} \tag{17}$$

When the singlet scalar fields acquire their v.e.v's, the $U(1)$ symmetry gets broken and the neutrinos obtain their Majorana masses [16]. One of the neutrino masses blows up, therefore, we need to introduce a Majorana mass term as a $U(1)$ symmetry breaking term, which is not going to change the mixing,

$$\mathcal{L}_M = \frac{1}{2} M [\bar{\nu}_{eR} \nu_{eR}^c + \bar{\nu}_{\mu R} \nu_{\mu R}^c + \bar{\nu}_{\tau R} \nu_{\tau R}^c] + h.c. \tag{18}$$

The most general scalar potential V that is invariant under the above symmetries is [13]

$$V = -\mu_1^2 |\epsilon_1|^2 - \mu_2^2 |\epsilon_2|^2 - \mu_3^2 |\epsilon_3|^2 + \lambda_1 |\epsilon_1|^4 + \lambda_2 |\epsilon_2|^4 + \lambda_3 |\epsilon_3|^4$$

$$\begin{aligned}
& + \varsigma_1 |\epsilon_1|^2 |\epsilon_2|^2 + \varsigma_2 |\epsilon_1|^2 |\epsilon_3|^2 + \varsigma_3 |\epsilon_2|^2 |\epsilon_3|^2 + \sigma_1 |\epsilon_1|^2 |\phi_1|^2 + \sigma_2 |\epsilon_1|^2 |\phi_2|^2 \\
& + \sigma_3 |\epsilon_2|^2 |\phi_1|^2 + \sigma_4 |\epsilon_2|^2 |\phi_2|^2 + \sigma_5 |\epsilon_3|^2 |\phi_1|^2 + \sigma_6 |\epsilon_3|^2 |\phi_2|^2 + V_{2HD}(\phi_1, \phi_2) \quad (19)
\end{aligned}$$

where $V_{2HD}(\phi_1, \phi_2)$ is the potential of the two Higgs doublets,

$$\begin{aligned}
V_{2HD}(\phi_1, \phi_2) &= -\mu_{\phi_1}^2 \phi_1^\dagger \phi_1 - \mu_{\phi_2}^2 \phi_2^\dagger \phi_2 + \lambda_{\phi_1} (\phi_1^\dagger \phi_1)^2 + \lambda_{\phi_2} (\phi_2^\dagger \phi_2)^2 + \lambda_{\phi_{12}} \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \right)^2 \\
&+ \lambda'_{\phi_{12}} \left(\phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2 \right)^2 + \lambda_{\phi_{21}} \left((\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \right) \\
&+ \lambda'_{\phi_{21}} \left((\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \right). \quad (20)
\end{aligned}$$

If we impose an additional S_3 symmetry on the three singlet scalar fields in the effective potential, it will be give by

$$\begin{aligned}
V &= -\mu^2 (|\epsilon_1|^2 + |\epsilon_2|^2 + |\epsilon_3|^2) + (|\epsilon_1|^2 + |\epsilon_2|^2 + |\epsilon_3|^2) \sum_{i=1}^2 \sigma_i \phi_i^\dagger \phi_i \\
&+ \lambda (|\epsilon_1|^2 + |\epsilon_2|^2 + |\epsilon_3|^2)^2 + V_{2HD}(\phi_1, \phi_2). \quad (21)
\end{aligned}$$

One can easily verify that the v.e.v's of the Higgs doublets are different and non-zero in the symmetric limit [13].

We can minimize the potential to get the v.e.v's ($\langle 0 | \epsilon_k^0 | 0 \rangle = w_k$) as follows

$$\begin{aligned}
\left. \frac{\partial V}{\partial |\epsilon_1|} \right|_{\min} &= -2\mu^2 w_1 + 2w_1 \sum_{i=1}^2 \sigma_i v_i^\dagger v_i + 4\lambda w_1 (w_1^2 + w_2^2 + w_3^2) = 0, \\
\left. \frac{\partial V}{\partial |\epsilon_2|} \right|_{\min} &= -2\mu^2 w_2 + 2w_2 \sum_{i=1}^2 \sigma_i v_i^\dagger v_i + 4\lambda w_2 (w_1^2 + w_2^2 + w_3^2) = 0, \\
\left. \frac{\partial V}{\partial |\epsilon_3|} \right|_{\min} &= -2\mu^2 w_3 + 2w_3 \sum_{i=1}^2 \sigma_i v_i^\dagger v_i + 4\lambda w_3 (w_1^2 + w_2^2 + w_3^2) = 0. \quad (22)
\end{aligned}$$

One can notice that the three equations are not independent. Thus, we have the three v.e.v's are the same and equal to

$$w^2 = \frac{\mu^2 - (\sigma_1 |v_1|^2 + \sigma_2 |v_2|^2)}{6\lambda}, \quad (23)$$

where $w_k = w$ for $k = 1, 2, 3$. The explicit form of the Yukawa matrix, Majorana and Dirac neutrino mass matrices can be written from the Lagrangian (17) as follows

$$Y_{23}^L = \frac{v_1}{\sqrt{2}} \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & y_2 \\ 0 & y_2 & y_2 \end{pmatrix},$$

$$\begin{aligned}
M_R &= \begin{pmatrix} M + \sqrt{2}v_w & 2v_w & 0 \\ 2v_w & M & 0 \\ 0 & 0 & M \end{pmatrix}, \quad \text{with } v_w = y \frac{w}{\sqrt{2}} \\
M_D &= \text{diag}(A, A, A), \quad \text{with } A = y \frac{v_2}{\sqrt{2}}.
\end{aligned} \tag{24}$$

Using the seesaw formula [17], the neutrino mass matrix is given as

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D. \tag{25}$$

Then \mathcal{M}_ν has the structure

$$\mathcal{M}_\nu = \begin{pmatrix} X & G & 0 \\ G & Y & 0 \\ 0 & 0 & Z \end{pmatrix}, \tag{26}$$

where

$$\begin{aligned}
X &= -\frac{A^2 M}{M^2 + \sqrt{2}Mv_w - 4v_w^2}, \quad Y = -\frac{A^2(M + \sqrt{2}v_w)}{M^2 + \sqrt{2}Mv_w - 4v_w^2}, \\
G &= \frac{2A^2 v_w}{M^2 + \sqrt{2}Mv_w - 4v_w^2}, \quad Z = -\frac{A^2}{M}.
\end{aligned} \tag{27}$$

One can easily verify that the relation $G = \sqrt{2}(X - Y)$ in Eq. 13 is satisfied. The mass eigenvalues $(2X - Y, 2Y - X, Z)$ can be written as

$$\begin{aligned}
m_1 &= -\frac{A^2}{M + 2\sqrt{2}v_w}, \\
m_2 &= -\frac{A^2}{M - \sqrt{2}v_w}, \\
m_3 &= -\frac{A^2}{M}.
\end{aligned} \tag{28}$$

From the above equations one can estimate the range of the v.e.v, v_2 . As the absolute neutrino masses are in the eV scale, therefore, v_2 has to be in the MeV scale if the see-saw scale (M) is in the TeV range. The mass eigenvalues satisfy the relation

$$\frac{1}{m_1} + \frac{2}{m_2} = \frac{3}{m_3}. \tag{29}$$

Similar relations among the masses are discussed in Ref. [18]. Thus, we can use the above sum-rule to obtain an upper limit for the heaviest mass $|m_3| \leq \frac{3|m_1||m_2|}{|2|m_1|+|m_2||}$ for the normal hierarchy or $|m_2| \leq \frac{2|m_1||m_3|}{|3|m_1|-|m_3||}$ for the inverted hierarchy.

4 Symmetry Breaking

The breaking of flavor symmetries in the charged lepton and neutrino sector cause deviations from the TBM form. Symmetry breaking in the charged lepton sector has been considered in Ref. [13]. They evaluated the correction angles numerically. In this section we are going to consider deviations of the TBM structure from the neutrino sector.

We are going to break the S_3 symmetry, which has led to equal v.e.v's in the symmetric limit, and maintain the symmetries of the Lagrangian. We will break the symmetry by introducing symmetry breaking terms of dimension four. We can present a large number of symmetry breaking terms. But for simplicity, we will shift the v.e.v of the two singlet scalars (ϵ_1, ϵ_2) to violate the decoupling in the neutrino mass matrix. Here, we introduce the most general form of symmetry breaking terms

$$\xi (|\epsilon_1|^2 - |\epsilon_2|^2)^2 + (|\epsilon_1|^2 - |\epsilon_2|^2) \sum_{i=1}^2 \rho_i \phi_i^\dagger \phi_i + \varrho (|\epsilon_1|^4 - |\epsilon_2|^4) \quad (30)$$

The most general symmetry breaking terms can be expressed in terms of the form in Eq. 30 and symmetry conserving terms that can be absorbed in the symmetric potential. Thus, the scalar potential including all the terms of the form in Eq. 30 is given as follows

$$\begin{aligned} V = & -\mu^2 (|\epsilon_1|^2 + |\epsilon_2|^2 + |\epsilon_3|^2) + (|\epsilon_1|^2 + |\epsilon_2|^2 + |\epsilon_3|^2) \sum_{i=1}^2 \sigma_i \phi_i^\dagger \phi_i + \lambda (|\epsilon_1|^2 + |\epsilon_2|^2 + |\epsilon_3|^2)^2 \\ & + \xi (|\epsilon_1|^2 - |\epsilon_2|^2)^2 + (|\epsilon_1|^2 - |\epsilon_2|^2) \sum_{i=1}^2 \rho_i \phi_i^\dagger \phi_i + \varrho (|\epsilon_1|^4 - |\epsilon_2|^4) \\ & + \xi' (|\epsilon_1|^2 + |\epsilon_2|^2)^2 + (|\epsilon_1|^2 + |\epsilon_2|^2) \sum_{i=1}^2 \rho'_i \phi_i^\dagger \phi_i + \varrho' (|\epsilon_1|^4 + |\epsilon_2|^4) + V_{2HD}(\phi_1, \phi_2). \end{aligned} \quad (31)$$

We can parametrize the v.e.v's of the singlet scalars as

$$\langle 0 | \epsilon_1 | 0 \rangle = \beta_1 \cos \gamma, \quad \langle 0 | \epsilon_2 | 0 \rangle = \beta_1 \sin \gamma, \quad \text{and} \quad \langle 0 | \epsilon_3 | 0 \rangle = \beta_2. \quad (32)$$

We require that all terms in the symmetry breaking potential are of the same size which results in, from Eq. 30, $\varrho \sim \frac{v^2}{\beta_1^2} \rho_i$ and $\xi \sim \frac{v^2}{\beta_1^2 \cos 2\gamma} \rho_i$ where $v^2 = v_1^2 + v_2^2$ is the EW scale. The only terms that depend on γ are

$$f(\gamma) = \xi \beta_1^4 \cos^2 2\gamma + \beta_1^2 \cos 2\gamma \sum_{i=1}^2 \rho_i |v_i|^2 + \varrho \beta_1^4 \cos 2\gamma + \varrho' \beta_1^4 \left(\frac{1 + \cos^2 2\gamma}{2} \right). \quad (33)$$

After minimizing the potential, one can get the parameters of v.e.v's as follows

$$\begin{aligned}
\cos 2\gamma &= -\frac{\varrho\beta_1^2 + (\rho_1|v_1|^2 + \rho_2|v_2|^2)}{(2\xi + \varrho')\beta_1^2}, \\
\beta_1^2 &= \frac{|v_1|^2(\varrho\rho_1 - \rho'_1(2\xi + \varrho')) + |v_2|^2(\varrho\rho_2 - \rho'_2(2\xi + \varrho'))}{-\varrho^2 + 2\xi(2\xi' + \varrho') + \varrho'(2\xi' + \varrho')}, \\
\beta_2^2 &= \frac{\beta_2'^2}{-2\lambda(-\varrho^2 + 2\xi(2\xi' + \varrho') + \varrho'(2\xi' + \varrho'))},
\end{aligned} \tag{34}$$

where

$$\begin{aligned}
\beta_2'^2 &\equiv -\mu^2(-\varrho^2 + 2\xi(2\xi' + \varrho') + \varrho'(2\xi' + \varrho')) \\
&+ |v_1|^2(\sigma_1(-\varrho^2 + 2\xi(2\xi' + \varrho') + \varrho'(2\xi' + \varrho')) + 2\lambda(\varrho\rho_1 - \rho'_1(2\xi + \varrho'))) \\
&+ |v_2|^2(\sigma_2(-\varrho^2 + 2\xi(2\xi' + \varrho') + \varrho'(2\xi' + \varrho')) + 2\lambda(\varrho\rho_2 - \rho'_2(2\xi + \varrho'))).
\end{aligned} \tag{35}$$

Then, we find the following relation is satisfied

$$\beta_2^2 + \beta_1^2 = 3w^2. \tag{36}$$

In the above equations, since $\varrho \sim \frac{v^2}{\beta_1^2}\rho_i$ that leads to $\cos 2\gamma \approx 0$ up to corrections of v^2/β_1^2 where we assume β_1 in the TeV range where v is the EW scale. Then

$$\langle 0|\epsilon_1|0\rangle \approx \langle 0|\epsilon_2|0\rangle \approx \frac{\beta_1}{\sqrt{2}}, \quad \langle 0|\epsilon_3|0\rangle \approx \beta_2. \tag{37}$$

Hence, the Majorana mass matrix remains has the same structure as in Eq. 24,

$$M_R = \begin{pmatrix} M + v_{\beta_2} & v_{\beta_1} & 0 \\ v_{\beta_1} & M & 0 \\ 0 & 0 & M \end{pmatrix}. \tag{38}$$

where $v_{\beta_i} = y\beta_i$.

Now, we consider in our analysis the first order correction to $\cos 2\gamma$ ($\cos 2\gamma \approx \tau$) where the symmetry breaking term is defined by

$$\tau \equiv -\frac{\varrho\beta_1^2 + (\rho_1|v_1|^2 + \rho_2|v_2|^2)}{(2\xi + \varrho')\beta_1^2}. \tag{39}$$

This leads to shifting the v.e.v's of the two singlet scalars ($\langle 0|\epsilon_1|0\rangle \neq \langle 0|\epsilon_2|0\rangle$) up to the first order of τ . Then, the Majorana neutrino mass matrix in Eq. 38 takes the form

$$M_R = \begin{pmatrix} M + v_{\beta_2} & v_{\beta_{1p}} & v_{\beta_{1n}} \\ v_{\beta_{1p}} & M & 0 \\ v_{\beta_{1n}} & 0 & M \end{pmatrix}, \tag{40}$$

where

$$\begin{aligned} v_{\beta_{1p}} &= \frac{y}{\sqrt{2}}(\langle 0|\epsilon_1|0\rangle + \langle 0|\epsilon_2|0\rangle), \\ v_{\beta_{1n}} &= \frac{y}{\sqrt{2}}(\langle 0|\epsilon_1|0\rangle - \langle 0|\epsilon_2|0\rangle). \end{aligned} \quad (41)$$

We write the v.e.v's of the singlet scalars after symmetry breaking as

$$\begin{aligned} \langle 0|\epsilon_1|0\rangle &= \frac{\beta_1}{\sqrt{2}}\left(1 + \frac{\tau}{2}\right), \\ \langle 0|\epsilon_2|0\rangle &= \frac{\beta_1}{\sqrt{2}}\left(1 - \frac{\tau}{2}\right), \end{aligned} \quad (42)$$

then

$$\begin{aligned} v_{\beta p} &= v_{\beta_1}, \\ v_{\beta n} &= \frac{\tau}{2}v_{\beta_1}. \end{aligned} \quad (43)$$

Note that, from the discussion below Eq. 32 and using Eq. 39 one finds

$$\xi \sim \frac{v^2}{\beta_1^2 \tau} \rho_i \sim \frac{\varrho}{\tau} \quad (44)$$

which leads to $\xi = 10\varrho$ for $\tau = 0.1$.

One finds that breaking the S_3 symmetry to generate different v.e.v's for the singlet scalars is not sufficient to break the almost degeneracy of (m_1, m_2) to satisfy the squared mass difference measurements. Therefore, we introduce an additional $U(1)$ symmetry breaking term,

$$\frac{1}{2}M_1 [\bar{\nu}_{\mu R}\nu_{\mu R}^c + \bar{\nu}_{\tau R}\nu_{\tau R}^c] + h.c. \quad (45)$$

Thus

$$M_R = \begin{pmatrix} M + v_{\beta_2} & v_{\beta_1} & \frac{\tau}{2}v_{\beta_1} \\ v_{\beta_1} & M' & 0 \\ \frac{\tau}{2}v_{\beta_1} & 0 & M' \end{pmatrix}, \quad (46)$$

where $M' = M + M_1$. Using the see-saw formula, the neutrino mass matrix is given by

$$\mathcal{M}_\nu = \begin{pmatrix} X' & G' & P' \\ G' & Y' & W' \\ P' & W' & Z' \end{pmatrix}, \quad (47)$$

where

$$\begin{aligned}
X' &= -\frac{4A^2M'}{4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2(4 + \tau^2)}, \\
Y' &= -\frac{A^2(4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2\tau^2)}{M'(4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2(4 + \tau^2))}, \\
Z' &= -\frac{4A^2(MM' + M'v_{\beta_2} - v_{\beta_1}^2)}{M'(4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2(4 + \tau^2))}, \\
G' &= \frac{4A^2v_{\beta_1}}{4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2(4 + \tau^2)}, \\
P' &= \frac{2A^2v_{\beta_1}\tau}{4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2(4 + \tau^2)}, \\
W' &= -\frac{2A^2v_{\beta_1}^2\tau}{M'(4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2(4 + \tau^2))}.
\end{aligned} \tag{48}$$

From Eqs. 47, one gets the mass eigenvalues

$$\begin{aligned}
m_1 &= -A^2 \frac{2(M + M' + v_{\beta_2}) - 2\sqrt{M^2 + M'^2 - 2M(M' - v_{\beta_2}) - 2M'v_{\beta_2} + v_{\beta_2}^2 + v_{\beta_1}^2(4 + \tau^2)}}{4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2(4 + \tau^2)}, \\
m_2 &= -A^2 \frac{2(M + M' + v_{\beta_2}) + 2\sqrt{M^2 + M'^2 - 2M(M' - v_{\beta_2}) - 2M'v_{\beta_2} + v_{\beta_2}^2 + v_{\beta_1}^2(4 + \tau^2)}}{4MM' + 4M'v_{\beta_2} - v_{\beta_1}^2(4 + \tau^2)}, \\
m_3 &= -A^2 \frac{1}{M'}.
\end{aligned} \tag{49}$$

We can diagonalize the mass matrix in Eq. 47 using the unitary matrix $U_\nu = W_{12}^\nu R_{23}^\nu R_{12}^\nu$ with,

$$\begin{aligned}
R_{12}^\nu &= \begin{pmatrix} c_{12\nu} & s_{12\nu} & 0 \\ -s_{12\nu} & c_{12\nu} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
c_{12\nu} &= \cos \theta_{12\nu}; s_{12\nu} = \sin \theta_{12\nu}, \\
R_{23}^\nu &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23\nu} & s_{23\nu} \\ 0 & -s_{23\nu} & c_{23\nu} \end{pmatrix}, \\
c_{23\nu} &= \cos \theta_{23\nu}; s_{23\nu} = \sin \theta_{23\nu}.
\end{aligned} \tag{50}$$

One can find relations between the mass matrix elements in Eq. 48

$$\begin{aligned}
X'(Z' - Y') &= P'^2 - G'^2, \\
G'P'(Z' - Y') &= W'(P'^2 - G'^2).
\end{aligned} \tag{51}$$

Applying the above relations to the corresponding mass matrix elements, $\mathcal{M}_\nu = U_\nu \mathcal{M}_\nu^d U_\nu^\dagger$ with $U_\nu = W_{12}^\nu R_{23}^\nu R_{12}^\nu$, one can get the two mixing angles

$$\begin{aligned} s_{23\nu} &= \sqrt{\frac{3m_1(m_2 - m_3)}{m_2(m_1 - m_3)}}, \\ s_{12\nu} &= \sqrt{\frac{-2m_1m_2 + 3m_1m_3 - m_2m_3}{3m_3(m_1 - m_2)}}. \end{aligned} \quad (52)$$

Following Ref. [19], we expand the angles in Eq. 1 as

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1 + s), \quad s_{23} = \frac{1}{\sqrt{2}}(1 + a), \quad (53)$$

where the three real parameters r, s, a describe the deviations of the reactor, solar, and atmospheric angles from their tri-bimaximal values. We use global fits of the conventional mixing parameters (s, a) [20] that can be translated into 3σ ranges and the mixing parameter r with 2.5σ significance (90% C.L.) [2]

$$0.12 < r < 0.39, \quad -0.13 < s < 0.05, \quad -0.15 < a < 0.16. \quad (54)$$

To first order in r, s, a the lepton mixing matrix can be written as [19],

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1 - \frac{1}{2}s) & \frac{1}{\sqrt{3}}(1 + s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1 + s - a + re^{i\delta}) & \frac{1}{\sqrt{3}}(1 - \frac{1}{2}s - a - \frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1 + a) \\ \frac{1}{\sqrt{6}}(1 + s + a - re^{i\delta}) & -\frac{1}{\sqrt{3}}(1 - \frac{1}{2}s + a + \frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1 - a) \end{pmatrix}. \quad (55)$$

We are not going to consider CP violation in this work, thus, we assume that $\delta = 0$. The present data prefers a negative value for s and r is positive [19]. We can write the parameters (r, s, a) in terms of the elements of the mixing matrix,

$$\begin{aligned} r &= -1 - s + a - \sqrt{6}U_{21}, \\ s &= -1 + \sqrt{3}U_{12}, \\ a &= -1 + \sqrt{2}U_{23}. \end{aligned} \quad (56)$$

Now, we can calculate the full deviation of the leptonic mixing coming from the charged lepton and neutrino sector. We obtain the elements of the lepton mixing matrix

$$U_{PMNS} = U_l^\dagger U_\nu, \quad (57)$$

with $U_\ell = W_{23}^l R_{23}^l R_{13}^l R_{12}^l$ and $U_\nu = W_{12}^\nu R_{23}^\nu R_{12}^\nu$. Thus, up to the first order one can get

$$\begin{aligned} r &\approx -s_{12l} + \sqrt{\frac{2}{3}}s_{23\nu} + s_{13l}, \\ s &\approx -s_{12l} + \sqrt{2}s_{12\nu} - s_{13l}, \\ a &\approx -s_{23l} + \sqrt{\frac{2}{3}}s_{23\nu}. \end{aligned} \quad (58)$$

In Ref. [13], it was found that the contribution of the charged lepton sector, with $\delta = 0$, is give as

- For $z = 2.0$: $s_{12l} \approx \pm 0.34$, $s_{13l} \approx \mp 0.0011$, $s_{23l} \approx -0.059$,
- For $z = 2.06$: $s_{12l} \approx \pm 0.3$, $s_{13l} \approx \mp 0.001$, $s_{23l} \approx -0.061$,
- For $z = 2.2$: $s_{12l} \approx \pm 0.2$, $s_{13l} \approx \mp 0.00075$, $s_{23l} \approx -0.065$,

where z is an arbitrary parameter with a value around 2. We can check the contributions of the charged leptons, $U_\ell = W_{23}^l R_{23}^l R_{13}^l R_{12}^l$, without corrections from the neutrino sector, $U_\nu = W_{12}^\nu$. By substituting the above values in Eq. 58, up to the first order one gets

- For $z = 2.0$: $r \approx 0.34$, $s \approx 0.34$, $a \approx -0.059$,
- For $z = 2.06$: $r \approx 0.30$, $s \approx 0.30$, $a \approx -0.061$,
- For $z = 2.2$: $r \approx 0.20$, $s \approx 0.20$, $a \approx -0.065$.

The above results do not match the experimental values where the charged lepton sector introduces a large correction to the mixing angle θ_{12} . Thus, it becomes necessary to combine the contributions come from the charged lepton and neutrino sector in order to calculate the full deviation from the TBM mixing.

5 Numerical results

In the case of degenerate neutrino masses $m_1 \approx m_2 \approx m_3$, one can find from Eq. 14 that $a \approx b \approx c$. This leads to a diagonalized neutrino mass matrix $\mathcal{M}_\nu \approx \text{diag}(a, a, a)$. That means the lepton mixing matrix does not include a contribution from the neutrino sector, which is inconsistent with the experimental data. Thus, in the symmetric limit our model excludes the case of the degenerate neutrino masses.

The numerics goes as following; we choose random values of the masses (m_1, m_2, m_3) which satisfy the experimental values of the squared mass differences

$$\begin{aligned} \Delta m_{21}^2 &= m_2^2 - m_1^2 = (7.59 \pm 0.20) \times 10^{-5} eV^2, \\ \Delta m_{32}^2 &= |m_3^2 - m_2^2| = (2.43 \pm 0.13) \times 10^{-3} eV^2. \end{aligned} \quad (59)$$

We substitute the masses in (r, s, a) in Eq. 56 with $(s_{12\nu}, s_{23\nu})$ given in Eq. 52 and $(s_{12l}, s_{23l}, s_{13l})$ in the previous section. If the results agree with the experimental constraints in Eq. 54 we plot, in Figs. (1, 2), the possible values of the absolute masses and the mixing angles. By substituting the masses that obtained above in Eq. 49, one can find the values of the Lagrangian parameters $(v_{\beta 1}, v_{\beta 2}, A, M, M')$. The results support the normal mass hierarchy. The figures show that the scale of the neutrino masses is in the few meV to ~ 50 meV range (meV = 10^{-3} eV). Also,

the θ_{13} agrees with the recent T2K measurement and the full contribution from both the charged lepton and neutrino sector accommodates θ_{12} . The graphs show that the see-saw scales (M, M') are in the TeV range, and the extra Higgs that generates the Dirac neutrino masses has v.e.v (v_2), included in A , in the MeV scale. Also, they indicate that the v.e.v's of the singlet scalar fields ($v_{\beta 1}, v_{\beta 2}$) are in the TeV scale. Various other mechanisms to generate the neutrino masses with TeV scale new physics are mentioned in Ref. [21].

Three mass-dependent neutrino observables are probed in different types of experiments. The sum of absolute neutrino masses $m_{cosm} \equiv \Sigma m_i$ is probed in cosmology, the kinetic electron neutrino mass in the beta decay (M_β) is probed in direct search for neutrino masses, and the effective mass for neutrinoless double beta decay (M_{ee}) such that the decay rate $\Gamma \propto M_{ee}^2$. In terms of the “bare” physical parameters m_i and $U_{\alpha i}$, the observables are given by [18]

$$\begin{aligned}\Sigma m_i &= |m_1| + |m_2| + |m_3|, \\ M_{ee} &= ||m_1||U_{e1}|^2 + |m_2||U_{e2}|^2 e^{i\zeta_1} + |m_3||U_{e3}|^2 e^{i\zeta_2}|, \\ M_\beta &= \sqrt{|m_1|^2|U_{e1}|^2 + |m_2|^2|U_{e2}|^2 + |m_3|^2|U_{e3}|^2}.\end{aligned}\tag{60}$$

where in our model we ignore the Majorana phases (ζ_1, ζ_2). We plot M_β versus Σm_i and M_{ee} versus m_{light} , where m_{light} is the lightest neutrino mass which is m_1 in this model. The graphs show that $\Sigma m_i \approx 0.06$ eV and $M_{ee} < M_\beta$ and $M_{ee} < 0.35$ eV [22].

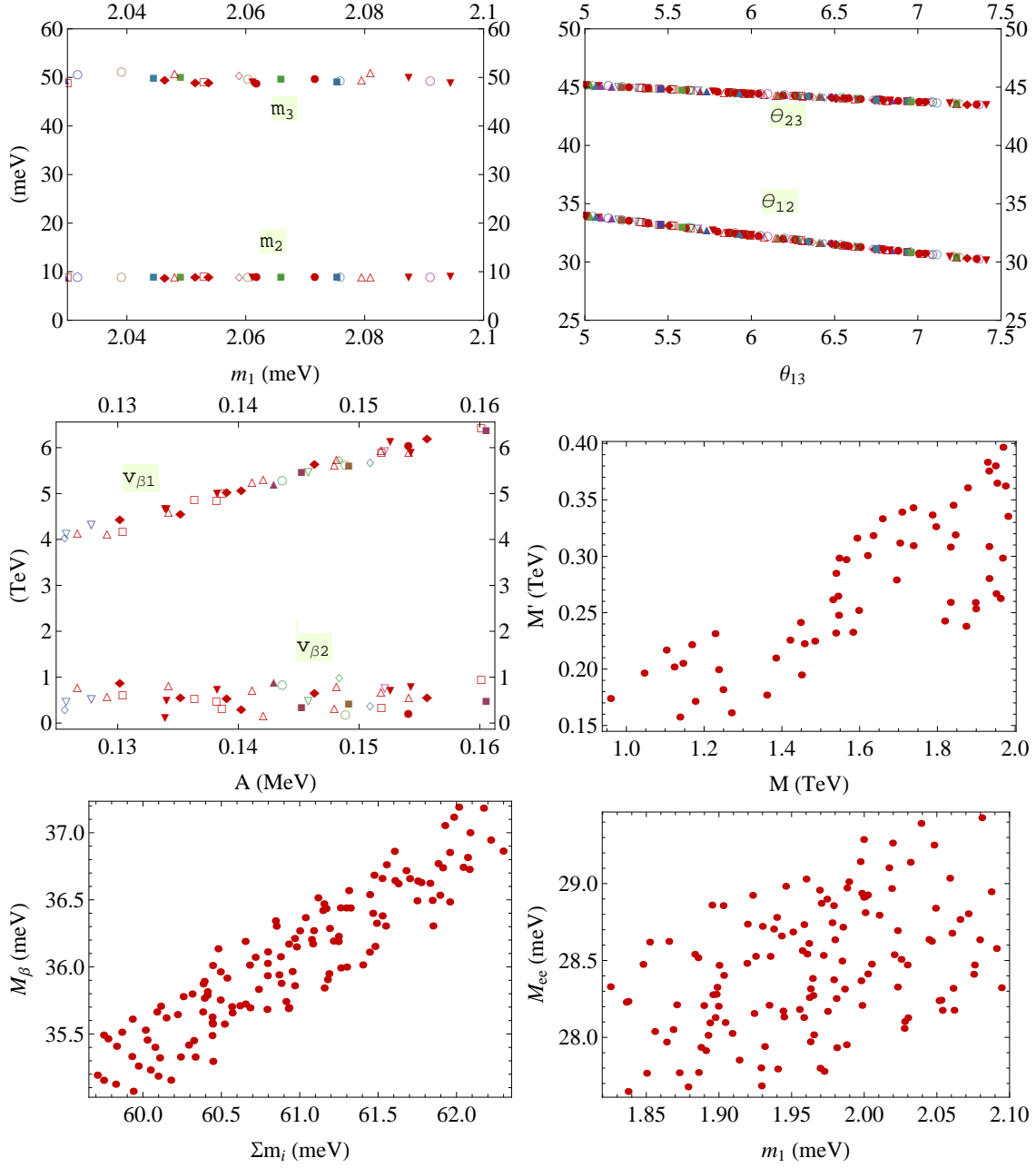


Figure 1: Scatter plot for $z = 2.0$ with $s_{12l} \approx -0.34$, $s_{13l} \approx 0.0011$, and $s_{23l} \approx -0.059$. In the neutrino sector, we take $\tau = 0.1$. (meV = 10^{-3} eV)

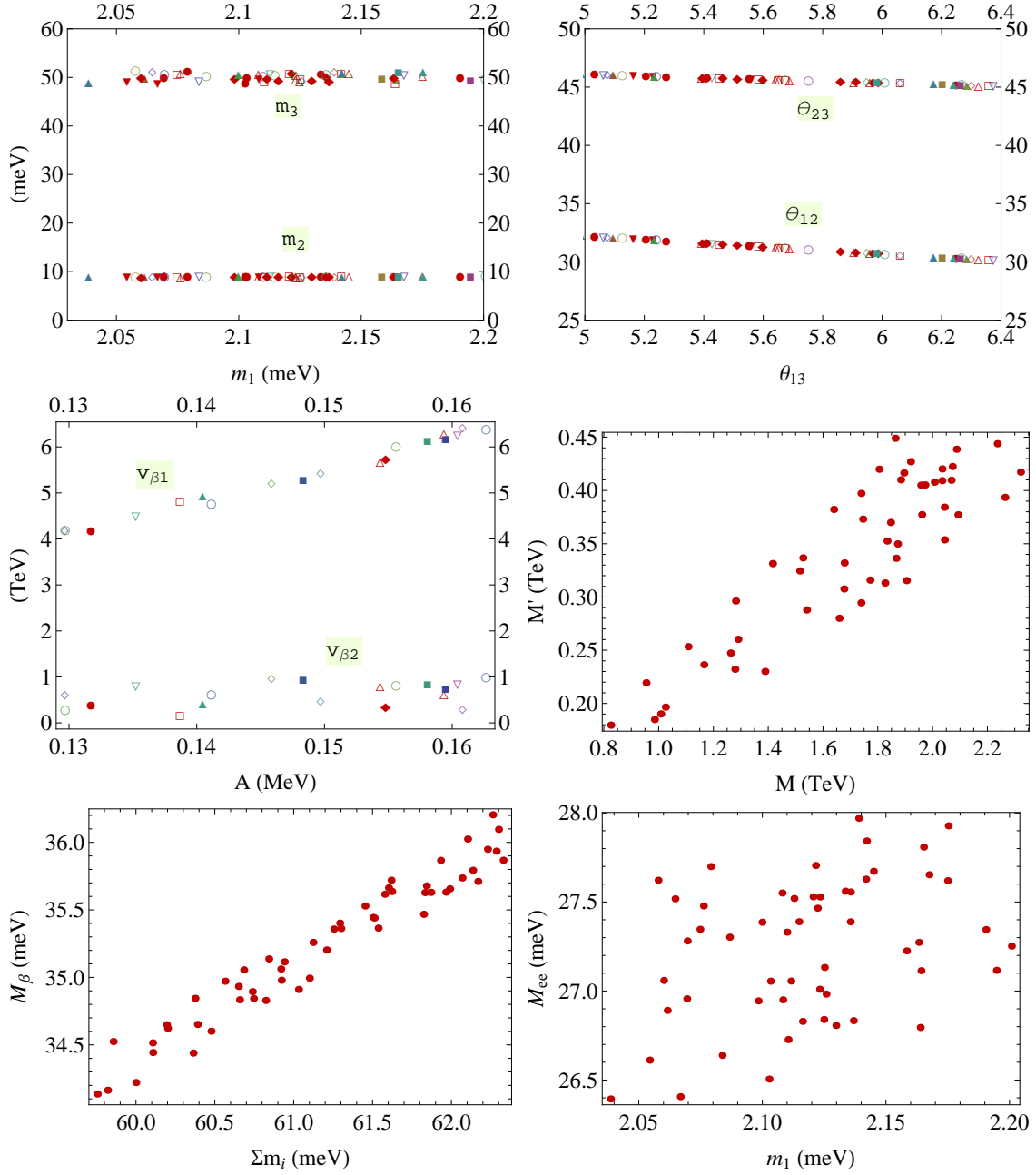


Figure 2: Scatter plot for $z = 2.06$ with $s_{12l} \approx -0.3$, $s_{13l} \approx 0.001$, and $s_{23l} \approx -0.061$. In the neutrino sector, we take $\tau = 0.05$. (meV = 10^{-3} eV)

6 Conclusion

In this paper, we extended our model in Ref. [13] to treat the leptonic mixing in the flavor symmetric limit as the tri-bimaximal pattern. The charged lepton sector was considered in Ref. [13] in a basis where the Yukawa matrix is non-diagonal with a $2 - 3$ symmetric structure except for one breaking by the muon mass. We fixed the neutrino mass matrix to have a decoupling of the first two generations from the third one, and under a certain condition we generated the lepton mixing with the TBM structure. This model was described by the Lagrangian that extended the SM by three right-handed neutrinos, an extra Higgs doublet, and three complex scalar fields. Also, the symmetry group of the SM was extended by the product of the symmetries $Z_2 \times Z_4 \times U(1)$.

The symmetry breaking in the charged lepton sector did not fix the data by introducing a large contribution to the mixing angle θ_{12} . Thereafter, by breaking the S_3 symmetry in the effective potential and violating the alignment of the v.e.v's of the singlet scalars, the contribution of the neutrino sector was introduced to accommodate the measurements. The analysis of our model to fit the experimental constraints of the mixing angles showed that this model supported the mass normal hierarchy with masses in the few meV to ~ 50 meV range. Also, the v.e.v of the additional Higgs was obtained in the MeV scale. The v.e.v of the singlet scalars and the see-saw scale were found to be in the TeV range. The graphs showed that $\Sigma m_i \approx 0.06$ eV and $M_{ee} < M_\beta$ and $M_{ee} < 0.35$ eV.

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